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Neural Extended Kalman Filter Based Angle-Only Target Tracking for Cruise Missiles

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Abstract— The main issue in the angle only target tracking is to estimate the states of a target by using noise corrupted measurement of elevation and azimuth. The tracking cruise missile is specified as a sea skimming anti-ship missile (SS-ASM) equipped with an active RF seeker. Although an active seeker gives the information of relative range to the target together with the line of side (LOS) angle and the LOS rate of elevation and azimuth, it cannot sense the relative range when the missile is jammed yet can still measure the LOS angles and LOS rates. The relative range information can be estimated by using the LOS angles and LOS rates in the jammed environment if the missile executes maneuver to ensure the observability for the range estimation. SS-ASMs fly below 5-10 meters over the sea and thus motion in elevation channel is not included through the estimation and it is assumed that the missile moves only in the horizontal plane. Another issue for SS-ASMs is the target velocity and the maneuverability profile. Constant velocity targets are examined due to fact that the ship targets are not capable of executing maneuver in a short time. Two different approaches for the range estimation are investigated and compared on the simulated data: the standard Extended Kalman Filter (EKF) and the Neural Extended Kalman Filter (NEKF). The system model for the estimation is formulated in terms of Modified Spherical Coordinates (MSC) for 2D horizontal missile-target geometry. Moreover, enhancement of the NEKF based estimation algorithm is given.

Keywords— Angle-only Target Tracking, Range Estimation, Neural Extended Kalman Filter, Cruise Missiles.

1 Introduction

Relative range information between a missile and a target can be used in terminal phase of the flight in order to increase the guidance performance of the missile [1]. In the case of the missiles equipped with a passive seeker or an active seeker but there is jammer in the environment, the range can be estimated if the missile maintains appropriate maneuvers to guarantee the observability.

The estimation of target position and velocity based on angle measurements is called angle-only target tracking, passive ranging or bearing-only-tracking. The fundamental of target tracking that is given in [2]. Ristic and Arulampalam [3] show and compare different types of tracking methods and coordinate systems. Cartesian Coordinates and Modified Spherical Coordinates (MSC) are usually used for target tracking.

Due to the fact that Cartesian coordinates are simple to implement, it is used extensively for target tracking with extended Kalman filter (EKF). In Cartesian coordinates system model is linear and measurement model is highly nonlinear. However, it is revealed that the filter with Cartesian coordinates shows unstable behavior characteristics [4]. In [5], the system is formulated in MSC which is well suited for angle-only target tracking.

Observability is the other issue in target tracking problem. Observability requirements are investigated for only the constant velocity trajectory case in two [6] and three dimensions [7]. Detailed works on observability can be found also in [8], [9], [10] and [11]. Also, implementation of pseudo linear filter for bearing-only target motion analysis can be found in [12] with the observability analysis.

Even if the full observability is obtained through the target state estimation problem, true states cannot be estimated exactly with standard EKF and error between the true states and the estimated states remains. At this point, NEKF can be used to eliminate this error. NEKF is introduced first in [13]. Main idea of the NEKF is to reduce effects of unmodeled dynamics, mismodeling, extreme nonlinearities and linearization in the standard EKF [14]. Using the NEKF instead of the EKF in the system model provides more accurate state estimate. Weights in the NEKF are coupled with the EKF states and the weights are trained by Kalman gains [15].

There are several areas of usage of NEKF. For instance, errors in sensor measurements may emerge from different sources such as noise and sensor limitations which may result in biases. In these cases calibration for the sensor model can be achieved by NEKF [16], [17]. Another area of usage is the tracking problems with interacting multiple models (IMM). The NEKF algorithm is used to improve motion model prediction during the target maneuver [18], [19], [20], [21], [22]. Moreover, NEKF is used for the missile intercept time calculation [23], [24].

2 Angle Only Target Tracking

2.1 Modified Spherical Coordinate

In MSC, x-axis points the east and y-axis points the north as demonstrated in Fig. 1. r denotes the range between the target and the observer. λ is the LOS angle and y-axis is along the initial LOS to the target so that $\lambda(0) = 0$.

For the angle-only target tracking, the states of analyzed filters are the reciprocal of the range, the LOS angle, the range rate divided by the range and the LOS rate.

$$y_{msc} = \begin{bmatrix} 1/r & \lambda & \dot{r}/r & \dot{\lambda} \end{bmatrix}^T \quad (1)$$

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$$y_{msc} = \begin{bmatrix} y_1 & y_2 & y_3 & y_4 \end{bmatrix}^T \quad (2)$$

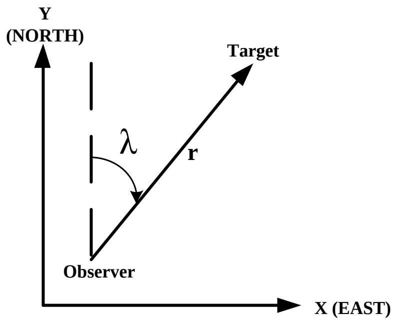

Fig. 1 Modified Spherical Coordinate

In MSC, the continuous state equation of motion for 2D intercept scenarios can be written as [5]

$$\begin{aligned} \dot{y}_1 &= -y_3 y_1 \\ \dot{y}_2 &= y_4 \\ \dot{y}_3 &= y_4^{\,2} - y_3^{\,2} + y_1\left[a_x \sin\left(y_2\right) + a_y \cos\left(y_2\right)\right] \\ \dot{y}_4 &= -2y_4 y_3 + y_1\left[a_x \cos\left(y_2\right) - a_y \sin\left(y_2\right)\right] \end{aligned} \quad (3)$$

where a_x and a_y are the Cartesian components of relative acceleration through north and east directions. For both stationary and constant velocity targets, acceleration components in (3) can be taken as negative of measured acceleration data of the missile because of the non-maneuvering target profile assumption. Moreover, MSC decouples the observable and unobservable components of state vector. Even if the filter is not fully observable, estimation performance of the observable states do not get affected from the unobservable states.

2.2 Observability Issue

Equation set (3) indicates that without any maneuver of the observer (a_x and a_y terms are equal to zero) the y_1 term drops off the $\dot{y}_3$ and the $\dot{y}_4$ functions. Therefore, y_1 (the reciprocal of the range) is not observable when neither the observer nor the target has any acceleration. In the case of stationary or constant velocity targets, the observer needs to execute maneuver for observability. However, it is stated that in [25], [26]; relative range can be estimated for stationary targets even if there is no observer maneuver for nonzero LOS rate condition. As a result, for stationary target state estimation problem, proposed equation of motion in [6] (also given in (2)) should be modified in such a way that the system becomes observable for stationary targets without any observer maneuver.

3 EKF AND NEKF ALGORITHMS

3.1 EKF

If the dynamic target model, relative states of the system and the measurements are taken into account, total system model for angle-only target tracking can be written as

$$\begin{aligned} \mathrm{x}_{k+1} &= f(\mathrm{x}_k, \mathrm{u}_k, \mathrm{w}_k) \\ \mathrm{y}_k &= c(\mathrm{x}_k, \mathrm{v}_k) \end{aligned} \quad (4)$$

where y is measurement vector, x is the state vector and u is the input vector. w and v represent the process and measurement noise respectively and they are assumed to be uncorrelated zero mean Gaussian noises with covariance matrices Q_k and R_k respectively.

$$\begin{aligned} \mathrm{w_k} &\sim \mathrm{N}\left(0, \mathrm{Q_k}\right) \\ \mathrm{v_k} &\sim \mathrm{N}\left(0, \mathrm{R_k}\right) \end{aligned} \quad (5)$$

One cycle of EKF algorithm is given below.

$$\left.\begin{aligned} \hat{x}_{k|k-1} &= f(\hat{x}_k, \mathrm{u}_k, \mathrm{w}_k) \\ \hat{P}_{k|k\text{-}1} &= J_f \hat{P}_{k\text{-}1|k\text{-}1} J_f^T + Q_k \\ K &= \hat{P}_{k|k\text{-}1} J_c^T \left[J_c \hat{P}_{k|k\text{-}1} J_c^T + R \right]^{-1} \\ \hat{x}_{k|k} &= \hat{x}_{k|k-1} + K\left[y_k - c(\hat{x}_{k|k-1})\right] \\ \hat{P}_{k|k} &= \hat{P}_{k|k-1} - K J_c \hat{P}_{k|k-1} \end{aligned}\right\} \quad (6)$$

where J_f and J_c are the Jacobeans of the functions f and c respectively.

3.2 NEKF

The NEKF is an estimation procedure that can be used in target tracking systems due to its adaptive nature [21]. When highly nonlinear systems are linearized and discretized or due to mismodeling of the system, the plant model may not be totally known [14]. When such conditions occur, estimation of the target states can become insufficient. The NEKF is used to compensate the unmodelled dynamics of the plant basically. A Neural network can be trained online with Kalman filter gains because the neural network weights are coupled to the standard EKF with the neural network terms [15].

The true system model is written as

$$x_{k+1} = f\left(x_k, u_k\right) \quad (7)$$

and the estimator system model is

$$\hat{x}_{k+1} = \hat{f}\left(\hat{x}_k, u_k\right) \quad (8)$$

The error between true and the estimated system $\epsilon = \mathrm{f} - \hat{\mathrm{f}}$ can be estimated by artificial neural network (ANN). Multi-layer perceptron (MLP) structure is used as ANN model. MLP consists of three or more layers (an input and an output layer with one or more hidden layers). A MLP with a single hidden layer scheme is given in Fig. 2. There are 4 neurons in the input and the output layer (one neuron for each state of the tracking filter). Also there are 3 neurons in the hidden layer. $\hat{y}_1$, $\hat{y}_2$, $\hat{y}_3$ and $\hat{y}_4$ are the estimated states of the tracking filter. After ANN modification, system becomes

$$f = \hat{f} + NN \quad (9)$$

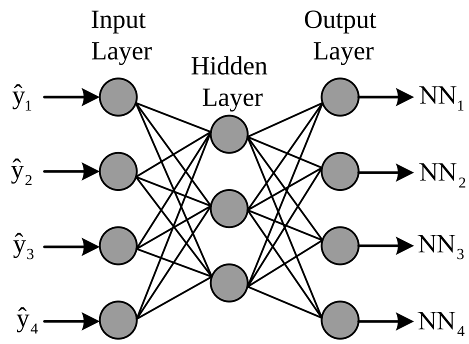

Fig. 2 ANN scheme for MLP

In the hidden layer of neural network, a large variety of functions can be used. The function usually used in NEKF is given below.

$$g(y) = \frac{1-e^{-y}}{1+e^{-y}} \tag{10}$$

Given activation function in (10) can squeeze the large magnitude values between -1 and +1. It is used as squashing function and shown in Fig. 3.

Each output of the ANN including the squashing function can be written as

$$NN_{k=1:4}(\mathrm{x}, w, \beta) = \sum_{j=1}^{3} \beta_{jk}\, g\left(\sum_{i=1}^{4} w_{ij} x_i\right) \tag{11}$$

where x_i's are the input signals to the neural network, in this case estimated states, the function 'g' is defined as activation function before, w and β are the input and the output weights of the neural network respectively. 'i' is the number of neurons in the input layer (4 for this case), 'j' is the number of neurons in the hidden layer (3 for this case), 'k' is the number of neurons in the output layer (4 for this case).

The NEKF is a combination of the EKF and neural network weights and so the NEKF state vector is

$$\bar{x}_k = \begin{bmatrix} x_k & w_k & \beta_k \end{bmatrix}^T \tag{12}$$

There are 4 states for the target tracking, 12 states for the input weights and 12 states for the output weights in the NEKF algorithm for this case and there are 28 states totally.

After including the ANN terms to the angle-only target tracking system, the implemented neural model of NEKF becomes

$$f(x_k, u_k) = \hat{f}(\hat{x}_k, u_k) + NN(\hat{x}_k, w_k, \beta_k) \tag{13}$$

The associated Jacobian of the NEKF for the target tracking would be

$$\bar{J}_f = \frac{\partial f(\bar{x}_k)}{\partial \bar{x}_k} = \begin{bmatrix} \underbrace{J_{11}}_{4x4} & \underbrace{J_{12}}_{4x24} \\ \underbrace{J_{21}}_{24x4} & \underbrace{J_{22}}_{24x24} \end{bmatrix}_{28x28} \tag{14}$$

where

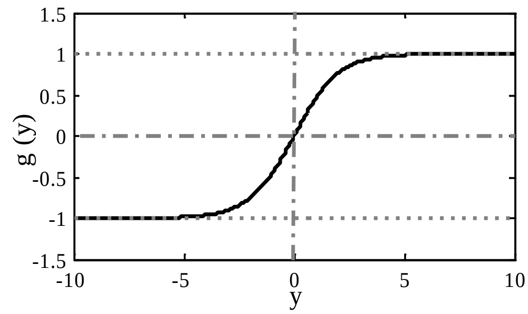

Fig. 3 Sigmoid squashing function

$$J_{11} = \left[\underbrace{\bar{A} + \frac{\partial NN(x_k, w_k, \beta_k)}{\partial x_k}}_{4x4}\right] \tag{15}$$

$$J_{12} = \left[\underbrace{\frac{\partial NN(x_k, w_k, \beta_k)}{\partial w_k}}_{4x12} \quad \underbrace{\frac{\partial NN(x_k, w_k, \beta_k)}{\partial \beta_k}}_{4x12}\right] \tag{16}$$

$$J_{21} = \left[\underbrace{0}_{24x4}\right] \tag{17}$$

$$J_{22} = \left[\underbrace{I}_{24x24}\right] \tag{18}$$

and $\bar{A}$ is the Jacobian of the continuous state equation of motion defined in (3). Finally, NEKF algorithm becomes

$$\left.\begin{aligned}
\hat{\bar{x}}_{k|k-1} &= \begin{bmatrix} f(\hat{x}_{k-1|k-1}) + NN(\hat{x}_{k-1|k-1}, \hat{w}_{k-1|k-1}, \hat{\beta}_{k-1|k-1}) \\ \hat{w}_{k-1|k-1} \\ \hat{\beta}_{k-1|k-1} \end{bmatrix} \\
\hat{P}_{k|k-1} &= \bar{J}_f \hat{P}_{k-1|k-1} \bar{J}_f^T + Q_k \\
K &= \hat{P}_{k|k-1} C^T \left[C \hat{P}_{k|k-1} C^T + R\right]^{-1} \\
\hat{\bar{x}}_{k|k} &= \begin{bmatrix} \hat{x}_{k|k} \\ \hat{w}_{k|k} \\ \hat{\beta}_{k|k} \end{bmatrix} = \hat{\bar{x}}_{k|k-1} + K\left[y_k - C\hat{\bar{x}}_{k|k-1}\right] \\
\hat{P}_{k|k} &= \hat{P}_{k|k-1} - KC\hat{P}_{k|k-1}
\end{aligned}\right\} \tag{19}$$

and for MSC target tracking problem, due to dimensionality measurement matrix (LOS and LOS rate are the measurements) in the above equation becomes

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 & \\ 0 & 0 & 0 & 1 & \end{bmatrix} 0_{2x24} \tag{20}$$

3.3 Process Noise Q and Measurement Noise R

The uncertainty in the state estimation due to random target dynamics or mismodeling of the target dynamics is typically represented by the process noise covariance matrix Q [27]. T is the sampling interval and σ_q is the target maneuver standard deviation. The choice of σ_q can be considered as tuning process for simulation results. For the states $\begin{bmatrix} x & y & \dot{x} & \dot{y} \end{bmatrix}^T$ process noise covariance matrix becomes in the Cartesian coordinates

$$Q = \begin{bmatrix} \frac{T^3}{3} & 0 & \frac{T^2}{2} & 0 \\ 0 & \frac{T^3}{3} & 0 & \frac{T^2}{2} \\ \frac{T^2}{2} & 0 & T & 0 \\ 0 & \frac{T^2}{2} & 0 & T \end{bmatrix} \sigma_q^2 \qquad (21)$$

Q is formulated in the Cartesian coordinates but the tracking filter is completed in MSC. To express process noise in MSC, transformation of Q from Cartesian to MSC is necessary [28].

The measurement noise in the LOS and the LOS rate are assumed to be independent. R is a diagonal matrix which is given below.

$$R = diag\begin{pmatrix} \sigma_\lambda^2 & \sigma_{\dot{\lambda}}^2 \end{pmatrix} \qquad (22)$$

where σ_λ and $\sigma_{\dot{\lambda}}$ are the standard deviation of the measurement noise.

4 Example Scenario

Results of the NEKF based angle-only target tracking simulation performed against constant velocity targets are given in this section. Before presenting the results, parameters should be set to initialize the filter. The first parameter is the initial range and taken as 24 km. Speed of the missile and the target is taken as 272 m/s and 30 m/s respectively. Standard deviation of the LOS angle measurement noise is 0.6 degree and 0.015 degree/s for the LOS rate measurement noise. Initial estimate of the range standard deviation is 2 km and standard deviation for the target speed is taken as 10 m/s. Sampling rate T is 0.01 s and σ_q is 0.01 m/s². For the NEKF, initial values of the weights are taken as 0.0001 and standard deviations of the weights are 10^{-10} I.

The flight paths of the missile-target and the corresponding interception geometry are demonstrated in Fig. 4. In this scenario, at the beginning the missile is at (0, 0) and the target is at (25,0) km. Also the missile has initial 4 degrees heading angle from the east. The target has constant 20 m/s velocity components through the east and the north. The missile is guided by proportional navigation guidance (PNG) until the relative range decreases up to 20 km. Then the missile starts sinusoidal motion by open-loop acceleration commands and

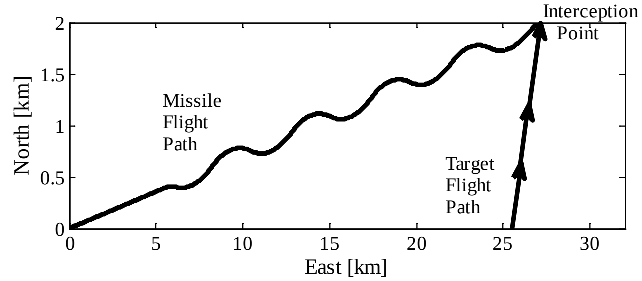

Fig. 4 Missile-target interception geometry

estimation starts with sinusoidal motion and ends with it. When the estimated relative range is less than 1 km, sinusoidal motion of the missile stops and missile is guided by PNG again at last 1 km.

In the evaluation, N=400 Monte Carlo simulations were performed with the same scenario but with different measurement noise characteristics. The performance is evaluated using the root mean square error for each time.

$$RMSE(t) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(x_i^{true} - \hat{x}_{t,i}\right)^2} \qquad (23)$$

In (23), $\hat{x}_{t,i}$ denotes the estimate at time t, for Monte Carlo simulation i. Simulation results are given in Fig. 5 to 8. Both the EKF and the NEKF give same results between the 20th and the 70th seconds for all states of the estimation. After the 70th second, the NEKF gives better result than the EKF. This situation stems from the fact that the LOS angle starts to grow through the end of the flight and this growth results with the linearization error in the Jacobian matrices due to small angle assumption and the neural terms of the NEKF tries to decrease the error in linearization. Basically, the NEKF improves the LOS angle filtering performance and this enhancement also improves the estimation of the other states. On the other hand, without the observability of the target, the NEKF cannot train its weights and weights are coupled to the estimation states so that the NEKF will diverge faster than the EKF. When designing the neural network based Kalman filter, the observability issue should be considered carefully.

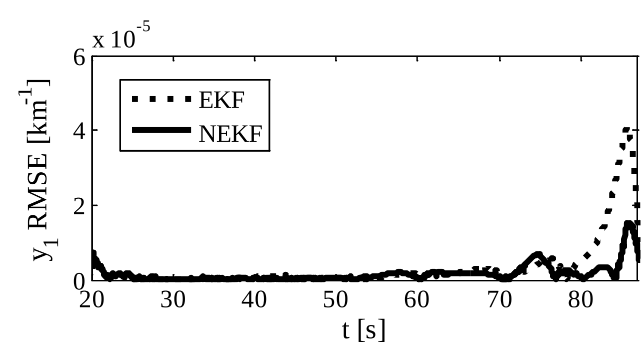

Fig. 5 RMSE in state $\mathbf{y_1}$

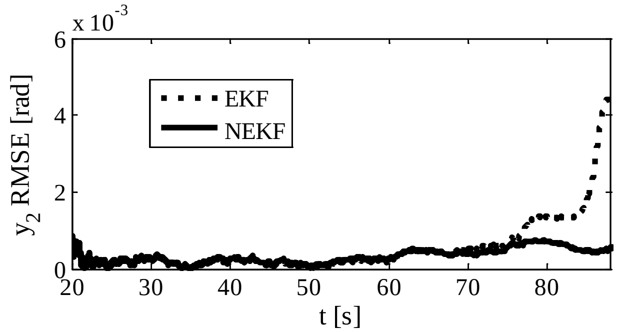

Fig. 6 RMSE in state $\mathbf{y_2}$

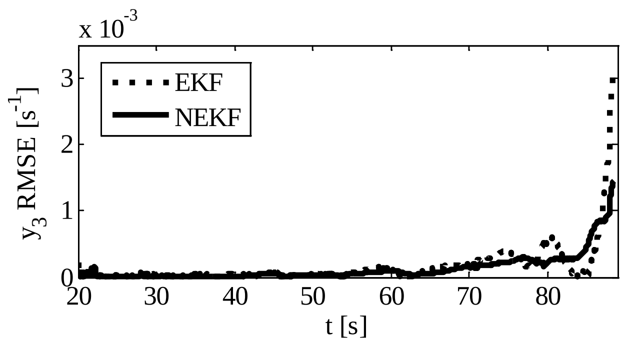

Fig. 7 RMSE in state $\mathbf{y_3}$

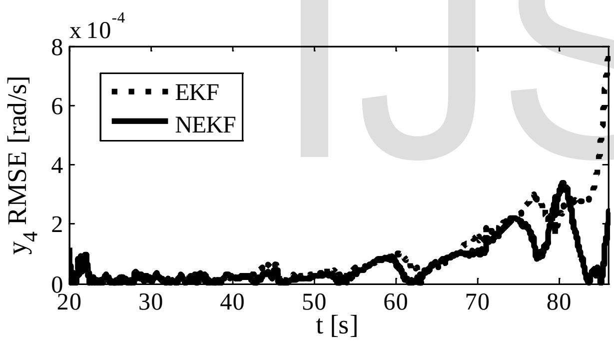

Fig. 8 RMSE in state $\mathbf{y_4}$

5 Conclusion

A neural network based EKF algorithm is analyzed in this paper. The NEKF has an adaptive nature and this nature is used to prevent estimation from the errors occurred due to the linearization or the mismodeling of the system. When sinusoidal motion is performed by the missile to enhance the observability, the LOS angle increases through the end of the flight and this situation causes a linearization error for the error covariance update of the estimation. As the LOS angle increases, the system moves off the linearization point and the filtering performance of the measurements decreases which also affects the estimation performance of the other states. In such cases, the NEKF proved that it compensated the unmodelled dynamics of the plant or the linearization error by learning online. The weights of the NN terms are trained with the EKF gains. The NEKF is shown to have improved performance over the EKF in relative range estimation of the angle-only target tracking.

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